Holographic entanglement and blocks in 2+1 flat space-times

Eliot Hijano (UBC)

Based on:

E.H. and C.Rabideau, JHEP 1805,068(2018) - arXiv:1712.07131

E.H. JHEP 1810,044(2018) - arXiv:1805.00949

E.H. and C.Rabideau, arxiv:18xx.xxxxx?

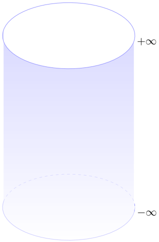
THE UNIVERSITY OF BRITISH COLUMBIA

It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

Some questions

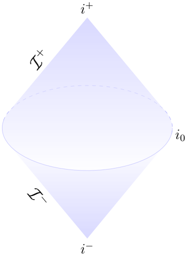
- Are the successes of holography contingent to AdS/CFT?
  - Do black holes in flat space-times evolve unitarily?
    [A. L. Fitzpatrick, J. Kaplan, D. Li, J. Wang]
  - Does the ETH hold in theories dual to gravity in flat space?
    [A. L. Fitzpatrick, J. Kaplan, M. T. Walters]
  - Can physics in flat-space be understood as emergent?
    [M. van Raamsdonk et all]
  - Can the physically sensible theories dual to gravity in flat space-times be classified through a bootstrap programme?
- In this talk we study the toy model of three-dimensional flat space-times.
  - Extrapolate dictionary
  - Entanglement entropy $=$ Geometry
  - Holographic blocks

Anti-de Sitter space / CFT



- A.S.G. $=$ Vir$\times$ Vir
- $\mathcal{O} = \lim\limits_{\rho \to \frac{\pi}{2}} (\cos\rho)^{\Delta}\, \Phi$
- Correlators $=$ Witten diagrams
- Conformal blocks $=$ Geodesic Witten diagrams
- $$S_{EE}(B) = \frac{\text{length}(\gamma_B)}{4G}$$

Flat space / BMSFT



- A.S.G. $=$ BMS$_3$
- $\mathcal{O} = \int d\lambda\, \Phi(\lambda)$
- Correlators $\sim$ Feynman diagrams
- BMS blocks $=$ "Geodesic Feynman diagrams"
- Entanglement $=$ Geometry ?

Asymptotic symmetry group at null infinity

- Global Minkowski metric is a solution of Einstein gravity with $\Lambda = 0$

$$ds^2 = -du^2 - 2dudr + r^2 d\phi^2$$

$u = t - r$ is the retarded time. $\mathcal{I}^+$ located at $r \to \infty$ with u, ϕ fixed.

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- Asymptotic symmetry group is generated by these diffeomorphisms

$$\xi^u = u\partial_\phi Y(\phi) + T(\phi)\,, \quad \xi^\phi = Y(\phi) - \frac{u}{r}\partial_\phi^2 Y(\phi) - \frac{1}{r}\partial_\phi T(\phi)\,, \quad \xi^r = -r\partial_\phi \xi^\phi\,.$$

with $T = e^{in\phi}$ and $Y = e^{in\phi}$. These diffeos form an algebra.

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with $T = e^{in\phi}$ and $Y = e^{in\phi}$. These diffeos form an algebra.

- Charges obey a centrally extended version of the algebra. BMS_3 algebra:

$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{m+n} + \frac{c_L}{12} n(n^2-1)\,, \quad [\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{M}_{n+m} + \frac{c_M}{12} n(n^2-1)\,,$$

$$[\mathcal{M}_n, \mathcal{M}_m] = 0\,, \quad \text{with} \quad c_L = 0 \quad \text{and} \quad c_M = \frac{3}{G}\,.$$

Representations

- There are two different kinds of representations of the BMS_3 algebra.

Highest weight representations	**Induced modules**
Non-relativistic limit of Vir×Vir highest weight representations	Ultra-relativistic limit of Vir× Vir highest weight representations
$L_0\|\Delta,\xi\rangle = \Delta\|\Delta,\xi\rangle\,, \quad M_0\|\Delta,\xi\rangle = \xi\|\Delta,\xi\rangle$ $M_{n>0}\|\Delta,\xi\rangle = L_{n>0}\|\Delta,\xi\rangle = 0$	$L_0\|M,s\rangle = s\|M,s\rangle\,, \quad M_0\|M,s\rangle = M\|M,s\rangle$ $M_{n\neq 0}\|M,s\rangle = 0$
Non-unitary	Unitary
State-Operator map in position space: $\|\Delta,\xi\rangle \sim \mathcal{O}(0)\|0\rangle$	State-Operator map in momentum space: $\|M,s\rangle \sim \mathcal{O}(p_0)\|0\rangle$

Aside: BMS_3 as a limit from Vir×Vir

The BMS_3 algebra can be obtained from two inequivalent limits of the algebra in a CFT_2

- Ultra-relativistic limit:

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}\,, \quad M_n = \epsilon\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$
$$c = \frac{c_L + c_M/\epsilon}{2}\,, \quad \bar{c} = \frac{-c_L + c_M/\epsilon}{2}$$
$$t \to \epsilon t\,, \quad x \to x$$

- Non-Relativistic limit:

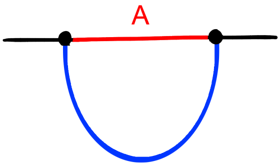
$$L_n = \mathcal{L}_n + \bar{\mathcal{L}}_n\,, \quad M_n = \epsilon\left(\mathcal{L}_n - \bar{\mathcal{L}}_n\right)$$
$$c = \frac{c_L + c_M/\epsilon}{2}\,, \quad \bar{c} = \frac{c_L - c_M/\epsilon}{2}$$
$$t \to t\,, \quad x \to \epsilon x$$

Results

- Holographic Entanglement
- Holographic Global BMS_3 blocks (highest weight representations)
- BMS_3 blocks (highest weight representations)
- Guess for an extrapolate dictionary

- BMS_3 blocks (induced modules) - Work in progress.

Reminder: Holographic Entanglement and Virasoro blocks in AdS_3/CFT_2



Perturbative heavy limit

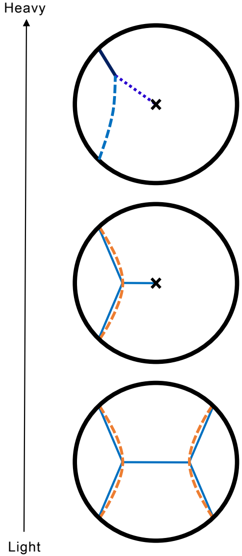
$h_{1,2} \sim c\ ,$

$1 \ll h_{3,4,p} \ll c$

Heavy-light limit

$h_{1,2} \sim c$

Global limit



Entanglement entropy

- Computing entanglement is a daunting task. Only possible when there is enough symmetry, like in a CFT. BMS_3 is infinite-dimensional, so one can hope to compute BMSFT entanglement. An approach is the "Rindler method".

Entanglement entropy

H. Jiang, W. Song, Q. Wen, JHEP07(2017)142 - 1706.07552

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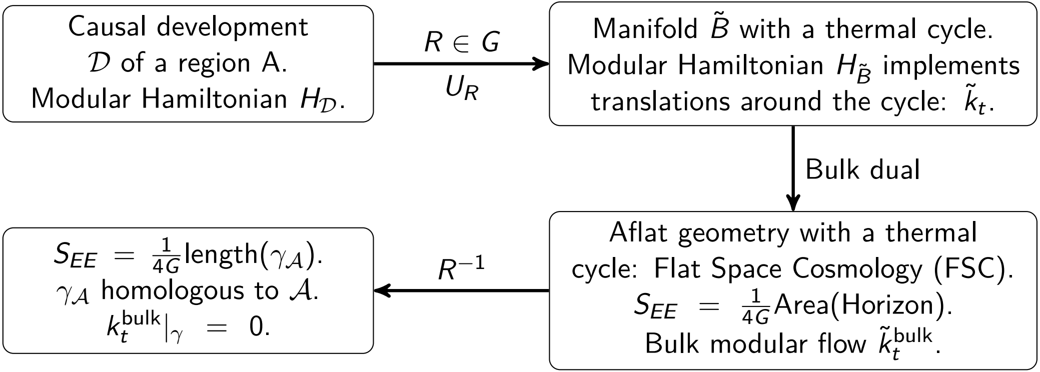
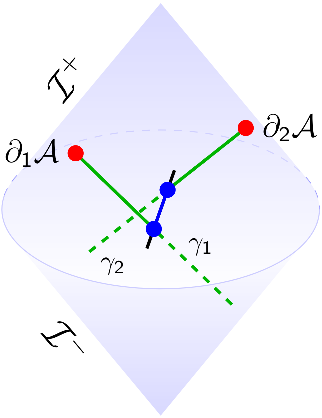
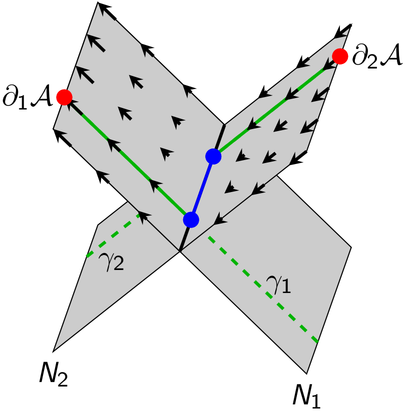
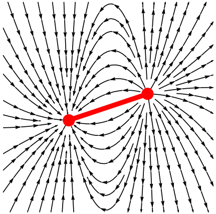
- Casini-Huerta-Myers implemented this method to derive the Ryu-Takayanagi formula.









$$S_{\text{EE}}(\mathcal{A}) = \frac{c_L}{6}\log\frac{x}{\epsilon} + \frac{c_M}{6}\frac{u}{x}\,.$$

Covariant approach to entanglement entropy

- Objective: Find a co-dimension two surface $\mathcal{S}$ homologous to $\mathcal{A}$, obeying $\delta\text{Area} = 0$.

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$$\varphi_1(x^\mu) = 0\,, \quad \text{and} \quad \varphi_2(x^\mu) = 0\,.$$

The extremality condition then reads

$$\delta\text{Area} = \int_{\mathcal{S}} \left(\theta_+ N_+^\mu \delta X_\mu + \theta_- N_-^\mu \delta X_\mu\right)$$

where $\theta_\pm$ are the "null expansions" that can be obtained from $\varphi_i(x^\mu)$.

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- Solving $\delta\text{Area} = 0$ implies solving $\theta_\pm = 0$, which yields explicit expressions for φ_i.
- In AdS_3 the solution for $\mathcal{S}$ is a geodesic anchored at $\partial\mathcal{A}$

$$S_{\text{EE}}^{\text{AdS}_3}(\mathcal{A}) = \frac{1}{4G}\text{Length}(\mathcal{S}) = \frac{1}{4G}\left(\log\frac{\omega_{12}}{\epsilon} + \log\frac{\bar{\omega}_{12}}{\epsilon}\right)\,.$$

A covariant approach in flat space

- We choose the constraints

$$\varphi_1(x^\mu) = u - U(r)\,, \quad \text{and} \quad \varphi_2(x^\mu) = \phi - \Phi(r)\,.$$

Solving $\theta_\pm = 0$ and $U, \Phi \to u_1, \phi_1$ as $r \to \infty$ yields

$$U(r) = u_1\,, \quad \text{and} \quad \Phi(r) = \phi_1\,, \quad \text{so } \mathcal{S} \text{ is a null line falling from } \partial_1\mathcal{A} \quad (\gamma_1).$$

Solving $\theta_\pm = 0$ for the curve starting at $\partial_2\mathcal{A}$ yields the null line falling from $\partial_2\mathcal{A}$ (γ_2).

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  Introduce a curve $\mathcal{S}'$ connecting the null lines at points $y_1, y_2 \in \gamma_1, \gamma_2$.
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- Minimal length geodesic yields the right result

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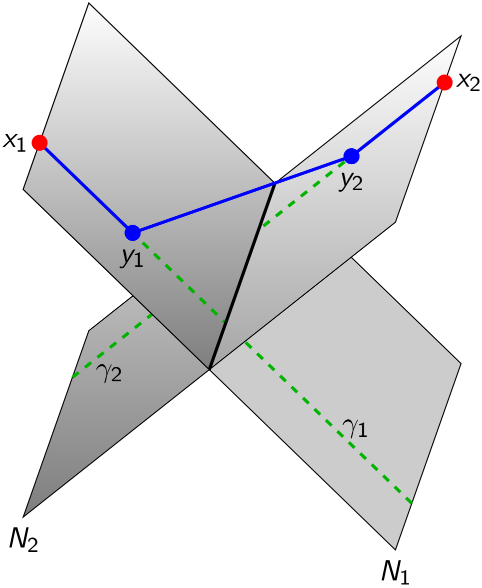
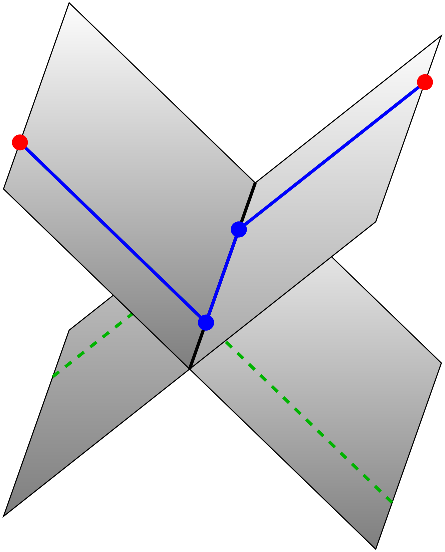

x_1
x_2
y_1
y_2
γ_1
γ_2
N_1
N_2



- Entanglement entropy in a theory with BMS_3 can be computed as

$$S_{\text{EE}} = -\lim_{n\to 1} \partial_n \text{Tr}\rho_{\mathcal{A}}^n = -\lim_{n\to 1} \partial_n \langle \Phi_n(\partial_1 \mathcal{A}) \Phi_n(\partial_2 \mathcal{A}) \rangle$$

where Φ_n are twist fields with $\xi_n = \frac{c_M}{24}(1 - \frac{1}{n^2})$

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- In the limit $n \to 1$, twist fields are "medium" operators.

$$1 << \xi_{n\to1} << c_M$$

In AdS/CFT, operators with this kind of quantum numbers correspond to probe particles in AdS_3, which move along geodesics.

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  In AdS/CFT, operators with this kind of quantum numbers correspond to probe particles in AdS_3, which move along geodesics.

- The holographic picture in flat space suggests that "medium" operators correspond to probe particles propagating in flat space-times, and sourced at null geodesics falling from the boundary.

Holographic three-point BMS correlators (probe limit)

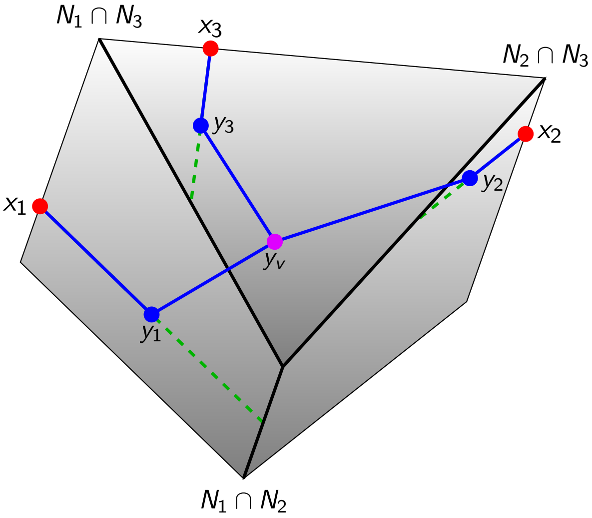
$$S_{\text{total}} = \sum_{k=1,2,3} \xi_k \left[L(x_k, y_k) + L(y_k, y_v) \right]$$

Extremize wrt y_v and $y_i \in \gamma_i$.
Solution is $y_v = N_1 \cap N_2 \cap N_3$.

$$\langle \Phi_{\xi_1} \Phi_{\xi_2} \Phi_{\xi_3} \rangle = e^{-S_{\text{total}}^{\text{Extremum}}}$$

$$= \prod_k e^{\xi_k \frac{\sum_{i<j} (-1)^{1+i+j} (u_i^\partial - u_j^\partial) \cos(\phi_k^\partial - \phi_i^\partial)}{\sum_{i<j} (-1)^{1+i+j} \sin(\phi_i^\partial - \phi_j^\partial)}} .$$

Matches correlator of primaries associated to highest weight representations of the BMS_3 algebra. $(\Delta = 0)$



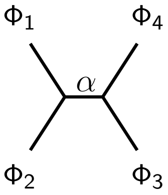
BMS$_3$ blocks (HW representations)

A. Bagchi, M. Gary, Zodinmawia - [1612.01730]

- Four-point correlators are not fixed by symmetry. Can be expanded in BMS-invariant blocks

$$\langle \Phi_\xi(x_1)\Phi_\xi(x_2)\Phi_\xi(x_3)\Phi_\xi(x_4)\rangle = \sum_i \langle \Phi_\xi(x_1)\Phi_\xi(x_2)|i\rangle\langle i|\Phi_\xi(x_3)\Phi_\xi(x_4)\rangle = \sum_{\mathcal{R}_\alpha} F_\alpha$$

$F_\alpha =$



$$\sim \frac{x^{\Delta_p - 2\Delta}}{\sqrt{1-x}}\left(1+\sqrt{1-x}\right)^{2-2\Delta_p} e^{2\xi\frac{t}{x} - \xi_p \frac{t}{x\sqrt{1-x}}}$$

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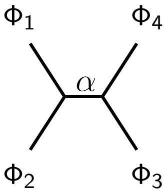

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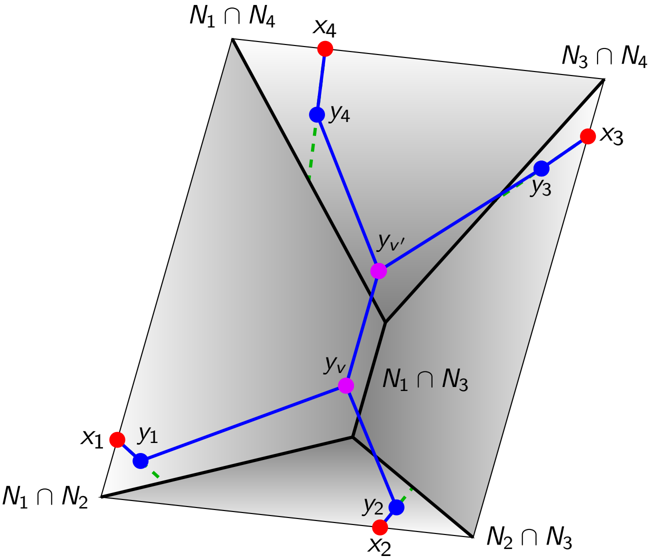
- Holographic calculation in the probe limit should correspond to a geodesic network attached to the four null lines falling from the boundary at the location of the operators. Very similar to AdS$_3$.


$N_1 \cap N_4$
x_4
$N_3 \cap N_4$
y_4
x_3
y_3
$y_{v'}$
y_v
$N_1 \cap N_3$
x_1
y_1
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y_2
x_2
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A proposal for the extrapolate dictionary

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- Proposal: Attach a position space Feynman diagram to the null lines and integrate the position of the legs over an affine parameter.

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\rangle \sim \int_{\gamma_{x_1}} d\lambda_1 \int_{\gamma_{x_2}} d\lambda_2 \ldots \langle \Psi_1(\lambda_1)\Psi_2(\lambda_2)\ldots\rangle \,.$$

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- Example: Three point function

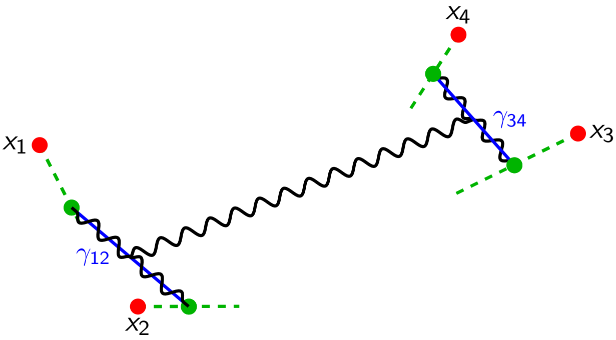
$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle \sim \int d^3y_v \prod_{i=1}^{3} \int_{\gamma_{x_1}} d\lambda_1 \int d^3p_i \frac{e^{ip_i\cdot(x_{\gamma_i}-y_v)}}{p_i^2+\xi_i^2-i\epsilon}$$

$$= \prod_k e^{\xi_k \frac{\sum_{i<j}(-1)^{1+i+j}(u_i^\partial - u_j^\partial)\cos(\phi_k^\partial-\phi_i^\partial)}{\sum_{i<j}(-1)^{1+i+j}\sin(\phi_i^\partial-\phi_j^\partial)}} \left(\sin\frac{\phi_i^\partial-\phi_j^\partial}{2}\right)^{-2} \,.$$

Geodesic Feynman Diagrams

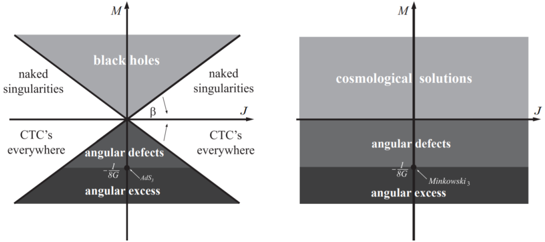
$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle \sim \int_{\gamma_{12}} ds \int_{\gamma_{34}} ds' \left(\prod_i \int_{\gamma_{x_i}} d\lambda_i\right)$$

$$G_F^{\xi}(\lambda_1,\sigma)\,G_F^{\xi}(\lambda_2,\sigma)\,G_F^{\xi_p}(\sigma,\sigma')\,G_F^{\xi}(\lambda_3,\sigma')\,G_F^{\xi}(\lambda_4,\sigma')\,.$$



BMS$_3$ blocks

- If operators are heavy: $\xi \sim c_M$, the fields in flat space should backreact the geometry. Q: Can Heavy-Heavy-Medium-Medium blocks be understood holographically as probe particles propagating in Asymptotically flat geometries?



$$ds^2 = Mdu^2 - 2dudr + Jdud\phi + r^2 d\phi^2 \,.$$

Thermal cycle

$$u \to u + \pi i \frac{J}{M}\,, \quad \phi \to \phi + \pi i \frac{2}{\sqrt{M}}\,.$$

G.Barnich, A.Gomberoff, H.A.González - [1204.3288]

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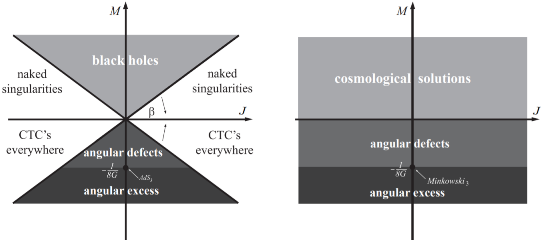

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G.Barnich, A.Gomberoff, H.A.González - [1204.3288]

- We will find a flat version of the ETH

$$M = \frac{24\xi_H}{c_M} - 1\,, \quad \text{and} \quad \frac{J}{2\sqrt{M}} = \frac{6\Delta_H}{c_M}\,.$$

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\begin{aligned}
L_0|\psi_1\rangle &= -\frac{1}{2}|\psi_1\rangle\,, \quad L_0|\psi_2\rangle = -\frac{1}{2}|\psi_2\rangle - \frac{9}{c_M}|\psi_1\rangle\,,\\
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- The multiplet is light in the large c_M limit.

$$\langle \mathcal{O}_1\mathcal{O}_2\psi_i\mathcal{O}_3\mathcal{O}_4\rangle = \Psi_i(x_i)\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle$$

where $\Psi_i(x_i)$ and its derivatives are $\mathcal{O}(e^{c_M^0})$.

- The multiplet obeys the following shortening conditions

$$\left(L_{-1}^2 + \frac{6}{c_M} M_{-2}\right) |\psi_1\rangle = 0$$

$$\left(L_{-1}^2 + \frac{6}{c_M} M_{-2}\right) |\psi_2\rangle + \frac{6}{c_M}\left(\frac{26}{c_M} M_{-2} + L_{-2}\right) |\psi_1\rangle = 0$$

These translate into differential equations

$$\left(\partial_u^2 + \frac{6}{c_M}\mathcal{M}\right) \psi_1 = 0\,,$$

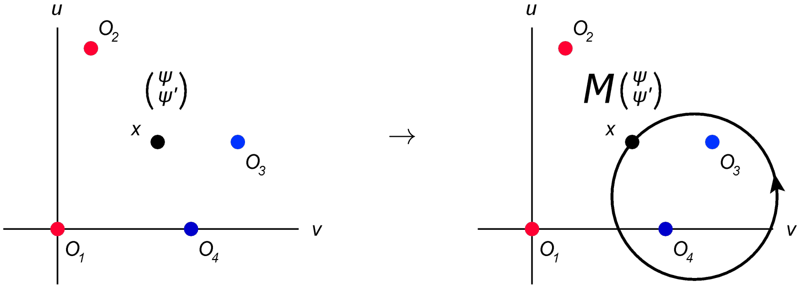
$$\left(\partial_u^2 + \frac{6}{c_M}\mathcal{M}\right) \psi_2 + \frac{6}{c_M}\left(\mathcal{L} + v\partial_u \mathcal{M}\right) \psi_1 = 0\,.$$

$\mathcal{M}$ and $\mathcal{L}$ can be obtained from BMS_3 ward identities

$$\mathcal{M}(x, x_i) = \sum_{i=1}^{n} \left(\frac{\xi_i}{(u-u_i)^2} + \frac{1}{u-u_i}\partial_{v_i}\right) \langle \mathcal{O}_1 \ldots \mathcal{O}_4 \rangle$$

$$\mathcal{L}(x, x_i) = \sum_{i=1}^{n} \left(\frac{\Delta_i}{(u-u_i)^2} - \frac{1}{u-u_i}\partial_{u_i} + 2\xi_i \frac{(v-v_i)}{(u-u_i)^3} + \frac{v-v_i}{(u-u_i)^2}\partial_{v_i}\right) \langle \mathcal{O}_1 \ldots \mathcal{O}_4 \rangle\,.$$

- The differential equations have a family of solutions that transform non-trivially when moved around pairs of operators. Fixing the monodromy effectively replaces pairs of operators by the contribution of a single conformal family in the OPE.



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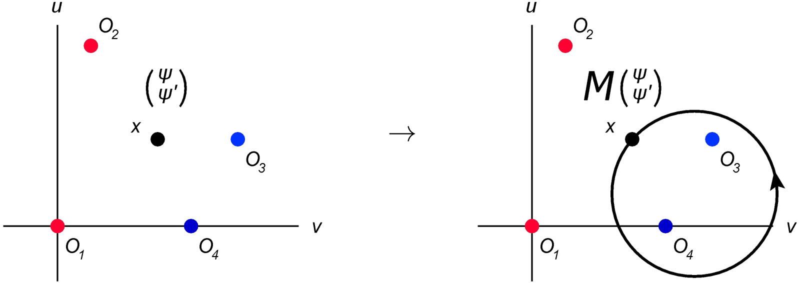
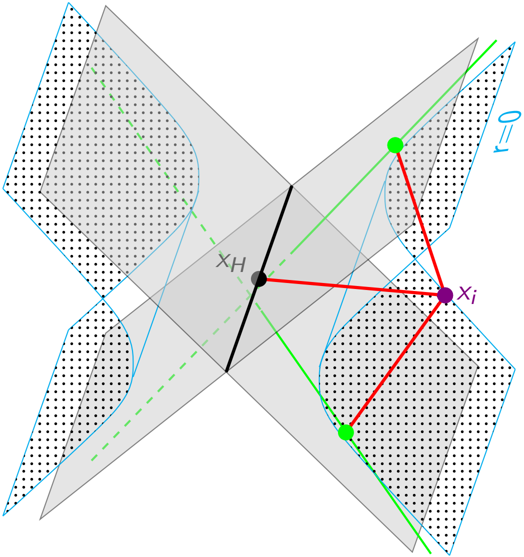


- Result:

$$\mathcal{F}_\alpha = \left(\frac{1-U^{\frac{\beta}{2}}}{1+U^{\frac{\beta}{2}}}\right)^{\Delta_\alpha} \left(\frac{U^{\beta-1}}{(1-U^\beta)^2}\right)^{\Delta_L}$$

$$\times\, e^{V\left(\frac{\beta U^{\frac{\beta}{2}}}{U(U^\beta-1)}\xi_\alpha - \frac{U^\beta(\beta+1)+\beta-1}{U(U^\beta-1)}\xi_L\right)+\Delta_H\left(\frac{2U^{\frac{\beta}{2}}}{\beta(U^\beta-1)}\xi_\alpha+\frac{2(U^\beta+1)}{\beta(1-U^\beta)}\xi_L\right)\log U} .$$

BMS$_3$ blocks - Bulk theory



Extremal geodesic network connecting the horizon of the Flat Space Cosmology with the null lines falling from infinity at the location of the "medium" operators.

Result matches the BMS$_3$ block obtained from the monodromy method if

$$M = \frac{24\xi_H}{c_M} - 1\,, \quad \text{and} \quad \frac{J}{2\sqrt{M}} = \frac{6\Delta_H}{c_M}\,.$$

A pure stated created by a heavy BMS$_3$ primary behaves like a thermal state at the level of correlation functions in the semi-classical limit (ETH).

Conclusions and future work

- Extrapolate dictionary between fields in flat space and BMS highest weight representations

$$\mathcal{O} \sim \int d\lambda \, \Phi(\lambda) \, .$$

- Some of what we know in AdS/CFT applies to flat space
  - Entanglement is geometric in the HRT sense.
  - Global blocks are geodesic networks (medium operators) or geodesic Feynman diagrams (light operators).
  - Heavy BMS primaries correspond to non-trivial geometries. Heavy enough operators behave thermally at the level of correlation functions (ETH).
- Unitary representations are more interesting. Global blocks read

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_4 \rangle \sim \delta^{(3)}(\sum_i p_i) \sum_\alpha C_{12\alpha} C_{34\alpha} \delta(s + M_\alpha^2) \, .$$

This can be understood from a flat limit of Virasoro global blocks.